Math 213, Spring 2018, Exam 3
Read Carefully, Write Neatly, Sketch Accurately

1. Evaluate $\iint_R\ ye^{xy}dA$, where R is the region bounded by $x=0$, $x=\frac{1}{y}$, $y=1$, $y=10$.

   $\iint_R\ ye^{xy}dA=\int_1^{10}\int_0^{\frac{1}{y}}ye^{xy}dx\,dy=\int_1^{10}[e^{xy}]_0^{\frac{1}{y}}dy=\int_1^{10}(e-1)dy=9(e-1)$

2. Sketch the region of integration, reverse the order of integration, and evaluate the integrals.

   a. $\int_0^8\int_{x^{\frac{1}{3}}}^2\frac{1}{y^4+1}dy\,dx$

   $\int_0^8\int_{x^{\frac{1}{3}}}^2\frac{1}{y^4+1}dy\,dx=\int_0^2\int_0^{y^3}\frac{1}{y^4+1}dx\,dy=\int_0^2\left[\frac{x}{y^4+1}\right]_0^{y^3}dy=\int_0^2\frac{y^3}{y^4+1}dy$

   $=\left[\frac{1}{4}\ln(y^4+1)\right]_0^2=\frac{1}{4}\ln 17$

   b. $\int_0^{2\sqrt{\ln 3}}\int_{\frac{y}{2}}^{\sqrt{\ln 3}}e^{x^2}dx\,dy$

   $\int_0^{2\sqrt{\ln 3}}\int_{\frac{y}{2}}^{\sqrt{\ln 3}}e^{x^2}dx\,dy=\int_0^{\sqrt{\ln 3}}\int_0^{2x}e^{x^2}dy\,dx=\int_0^{\sqrt{\ln 3}}\left[ye^{x^2}\right]_0^{2x}dx$

   $=\int_0^{\sqrt{\ln 3}}\left[2xe^{x^2}\right]dx=\left[e^{x^2}\right]_0^{\sqrt{\ln 3}}=3-1=2$

3. Sketch the region described, and find its area by evaluating an appropriate double integral.

   a. Region bounded by $y=1-x$, $y=2$, $y=e^x$.

   $A=\iint_R\ 1dA=\int_1^2\int_{1-y}^{\ln y}1dx\,dy=\int_1^2[\ln y-(1-y)]dy$

   $=\left[y\ln y-y-y-\frac{1}{2}y^2\right]_1^2=2\ln 2-\frac{1}{2}$

   b. Region bounded by $y=x-2$, $y=-x$, $y=\sqrt{x}$.

   $A=\iint_R\ 1dA=\int_0^1\int_{-x}^{\sqrt{x}}1dy\,dx+\int_1^4\int_{x-2}^{\sqrt{x}}1dy\,dx=\frac{13}{3}$

4. Evaluate $\iint_R\ \frac{\ln(x^2+y^2)}{x^2+y^2}dA$, where R is the region between two origin-centered circles, one of radius 1, the other of radius e.

   $\iint_R\ \frac{\ln(x^2+y^2)}{x^2+y^2}dA=\int_0^{2\pi}\int_1^e\frac{\ln(r^2)}{r^2}rdr\,d\theta=\int_0^{2\pi}\int_1^e\frac{2\ln(r)}{r}dr\,d\theta=\int_0^{2\pi}[(\ln r)^2]_1^e d\theta$

   $=\int_0^{2\pi}1d\theta=2\pi$

5. Sketch the solid below $z=\sqrt{x^2+y^2}$ and above the xy-plane in the first octant, inside the cylinder $x^2+y^2=1$. Compute the volume of the solid with an appropriate double integral.

   $V=\iint_R\ zdA=\int_0^{\frac{\pi}{2}}\int_0^1 rrdr\,d\theta=\int_0^{\frac{\pi}{2}}\left[\frac{1}{3}r^3\right]_0^1d\theta=\int_0^{\frac{\pi}{2}}\frac{1}{3}d\theta=\frac{\pi}{6}$

6. Setup and evaluate a triple integral to find the volume of the region in the first octant bounded by the coordinate planes and the planes $x+z=1$ and $y+2z=2$.

   $V=\iiint_D\ 1dV=\int_0^1\int_0^{2x}\int_0^{1-x}dz\,dy\,dx+\int_0^1\int_{2x}^2\int_0^{1-\frac{1}{2}y}dz\,dy\,dx=\cdots=\frac{2}{3}$

7. Sketch the region and find its volume by evaluating a triple integral in the appropriate coordinate system.

a. Region cut from the solid $x^2 + y^2 \leq 1$ by the sphere $x^2 + y^2 + z^2 = 4$.

$V = \iiint_D 1dV = \int_0^{2\pi} \int_0^1 \int_{-\sqrt{4-r^2}}^{\sqrt{4-r^2}} dz\, r dr\, d\theta = \cdots = \frac{32\pi}{3} - 4\pi\sqrt{3}$

b. The smaller region cut from the solid sphere $\rho \leq 2$ by the plane $z = 1$.

$V = \iiint_D 1dV = \int_0^{2\pi} \int_0^{\frac{\pi}{3}} \int_{\sec\phi}^{2} \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta = \cdots = \frac{5\pi}{3}$